

3-9

Properties of Parallel Lines

GEO

OBJECTIVE: I can prove theorems about parallel lines To use properties of parallel lines to find angle measures



Warm-Up

Look at the map of streets in Clearwater, Florida. Nicholson Street and Cedar Street are parallel. Which pairs of angles appear to be congruent?

$\angle 8$ and $\angle 12$ are congruent

$\angle 6$ and $\angle 10$ are congruent

$\angle 5$ and $\angle 9$ are congruent

$\angle 7$ and $\angle 11$ are congruent

$\angle 8, \angle 6, \angle 12, \angle 10$
are congruent

$\angle 5, \angle 9, \angle 7, \angle 11$
are congruent



Essential Understanding

Essential Understanding The special angle pairs formed by parallel lines and a transversal are congruent, supplementary, or both.



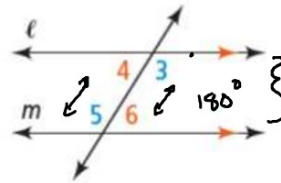
Key Concept: Same-Side Interior Angles Postulate

Postulate

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

If ...

$$\ell \parallel m$$



Then ...

$$m\angle 4 + m\angle 5 = 180$$

$$m\angle 3 + m\angle 6 = 180$$



Example

#1 Identifying Supplementary Angles



The measure of $\angle 3$ is 55. Which angles are supplementary to $\angle 3$? How do you know?

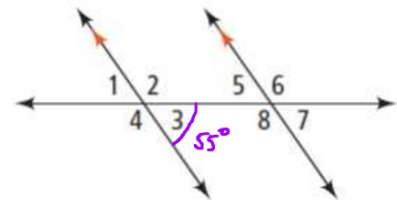
Supplementary, angles that add to 180°

$$\angle 3 + \angle 8 = 180^\circ$$

$$\angle 3 + \angle 2 = 180^\circ$$

$$\angle 3 + \angle 4 = 180^\circ$$

$$\angle 3 + \angle 6 = 180^\circ$$



Your Turn to Work it Out



1. **Reasoning** If you know the measure of one of the angles, can you always find the measures of all 8 angles when two parallel lines are cut by a transversal? Explain.

yes, by having one angle you can find the rest using corresponding vertical and supplementary angle rules.

Concept Understanding



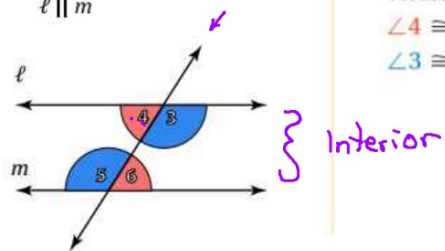
Key Concept: Alternate Interior Angles Theorem

Theorem

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

If...

$$\ell \parallel m$$



Then ...

$$\angle 4 \cong \angle 6$$

$$\angle 3 \cong \angle 5$$

Concept Understanding



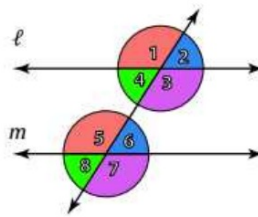
Key Concept: Corresponding Angles Theorem

Theorem

If a transversal intersects two parallel lines, then corresponding angles are congruent.

If...

$$\ell \parallel m$$



Then . . .

$$\angle 1 \cong \angle 5$$

$$\angle 2 \cong \angle 6$$

$$\angle 3 \cong \angle 7$$

$$\angle 4 \cong \angle 8$$

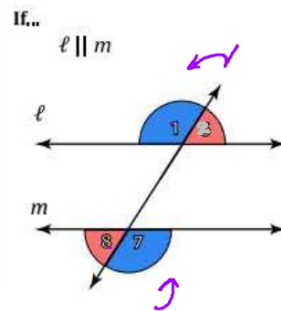
Concept Understanding



Key Concept: Alternate Exterior Angles

Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are congruent.



Then ...

$$\angle 1 \cong \angle 7$$

$$\angle 2 \cong \angle 8$$

Example

#2 Finding Measures of Angles



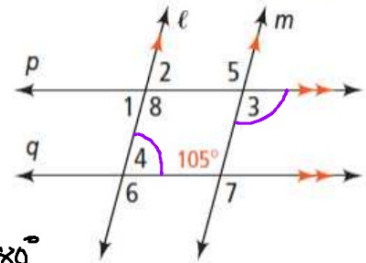
What are the measures of $\angle 3$ and $\angle 4$?

$\angle 3$ is the alternate interior angle to 105°

$\angle 3$ is 105°

$\angle 4$ is same-side interior angle that adds to 180°

$\angle 4$ is 75°



Your Turn to Work it Out



3. What is the measure of each angle?

a. $\angle 1 = 75^\circ$

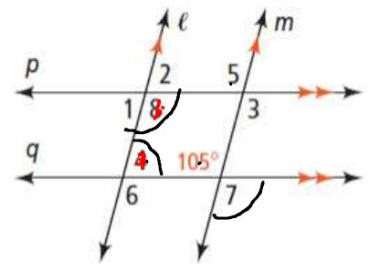
b. $\angle 2 = 75^\circ$

c. $\angle 5 = 105^\circ$

d. $\angle 6 = 105^\circ$

e. $\angle 7 = 105^\circ$

f. $\angle 8 = 105^\circ$



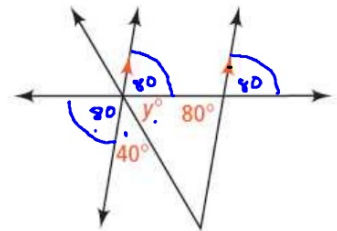
Example

#4 Finding an Angle Measure



Algebra What is the value of y ?

$$\begin{aligned}80^\circ + 40^\circ + y &= 180^\circ \\120^\circ + y &= 180^\circ \\ \underline{-120} \quad \quad \quad \underline{-120} & \\ y &= 60^\circ\end{aligned}$$



Your Turn to Work it Out



4.
a. In the figure at the right, what are the values of x and y ?

$$2x + x - 12 = 180^\circ$$

$$3x - 12 = 180^\circ$$

$$\begin{array}{r} +12 \quad +12 \\ \hline \end{array}$$

$$\frac{3x}{3} = \frac{192}{3}$$

$$x = 64$$

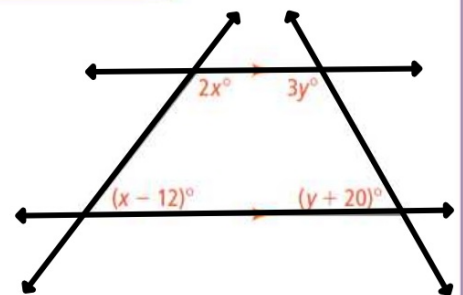
$$3y + y + 20 = 180^\circ$$

$$4y + 20 = 180^\circ$$

$$\begin{array}{r} -20 \quad -20 \\ \hline \end{array}$$

$$\frac{4y}{4} = \frac{160}{4}$$

$$y = 40$$



- b. What are the measures of the four angles in the figure?

$2x$	$x - 12$	$3y$	$y + 20$
$2(64)$	$64 - 12$	$3(40)$	$40 + 20$
128°	52°	120°	60°