## 5-2  Direct Variation

**OBJECTIVE:** I can write and graph an equation of a direct variation

## Warm-Up

The diagram shows how long it takes to hear thunder after you see lightning. What general rule can you use to model this situation? Explain.



10 s ÷ 5 2 mi

15 s ÷ 5 3 mi

$$y = \frac{x}{5}$$

## Essential Understanding

**Essential Understanding** If the ratio of two variables is constant, then the variables have a special relationship, known as a direct variation.

**Example** #1 Identifying a Direct Variation

A **direct variation** is a relationship that can be represented by a function in the form $y = kx$, where $k \neq 0$. The constant of variation for a direct variation k is the coefficient of x. By dividing each side of $y = kx$ by x, you can see that the ratio of the variables is constant: $\frac{y}{x} = k$.

To determine whether an equation represents a direct variation, solve it for y. If you can write the equation in the form $y = kx$, where k 0, it represents a direct variation.

**A** $7y = 2x$

$$\frac{7y}{7} = \frac{2x}{7}$$

$$y = \frac{2}{7}x$$

$\frac{2}{7}$ is the constant of variation

So this function is a direct variation

**B** $3y + 4x = 8$

$$3y + 4x = 8$$
$$\underline{-4x \quad -4x}$$
$$3y = -4x + 8$$

$$\frac{3y}{3} = \frac{-4x}{3} + \frac{8}{3}$$

$$y = -\frac{4x}{3} + \frac{8}{3}$$

This function cannot be written as
$$y = kx$$

1. Does 4x + 5y = 0 represent a direct variation? If so, find the constant of variation.

$$4x + 5y = 0$$

$$-4x \qquad -4x$$

$$5y = -4x$$

$$\frac{5y}{5} = \frac{-4x}{5}$$

$$y = -\frac{4}{5}x$$

Make the function look like

$$y = Kx$$

← This is a direct variation

the constant variation is $-\frac{4}{5}$

**Example** #2 Writing a Direct Variation Equation

To write an equation for a direct variation, first find the constant of variation k using an ordered pair, other than (0, 0), that you know is a solution of the equation.

$Y = Kx$

Suppose y varies directly with x and y = 35 when x = 5. What direct variation equation relates x and y? What is the value of y when x = 9?

STEP 1 → $y = Kx$ ← look for K

$(5, 35)$ $(35) = K(5)$

$\dfrac{35}{5} = \dfrac{5K}{5}$

← I can now write the direct variation rule

$7 = K$

$y = 7x$

STEP 2 → $y = 7x \; ; \; x = 9$ ← Format indicating substitute x in the function with the value 9

$y = 7(9)$

$y = 63$

2. Suppose y varies directly with x, and y = 10 when x = −2. What direct variation equation relates x and y? What is the value of y when x = −15?

$$y = Kx$$

$$(10) = K(-2)$$

$$\frac{10}{-2} = \frac{-2K}{-2}$$
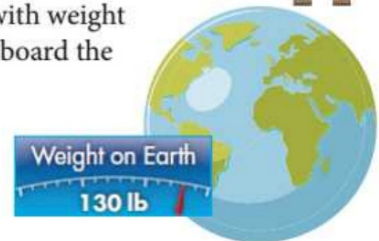
$$-5 = K$$

$$y = -5x$$

$$y = -5x$$

$$y = -5(-15)$$
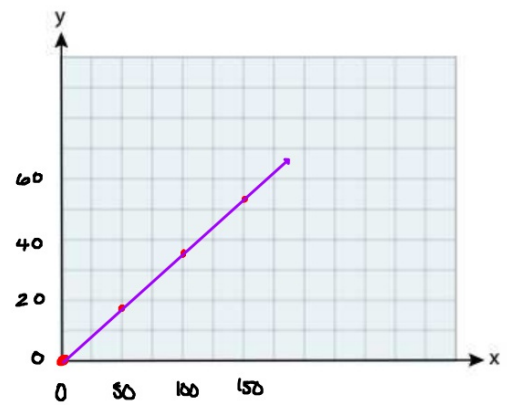
$$y = 75$$

#3 Graphing a Direct Variation

**Space Exploration** Weight on Mars y varies directly with weight on Earth x. The weights of the science instruments onboard the Phoenix Mars Lander on Earth and Mars are shown.

Weight on Mars
**50 lb**

Weight on Earth
**130 lb**

| X | Y |
|---|---|
| 0 | 0.38 (0) = 0 ✓ |
| 50 | 0.38 (50) = 19 |
| 100 | 0.38 (100) = 38 |
| 150 | 0.38 (150) = 57 |

$$y = Kx$$

$$\frac{(50)}{130} = \frac{x \, (130)}{130}$$

$$0.38 = K$$

$$y = 0.38 \, x$$

3. Weight on the moon ⟨y varies directly with weight on Earth x⟩ A person who weighs 100 lb on Earth weighs 16.6 lb on the moon. What is an equation that relates weight on Earth x and weight on the moon y? What is the graph of this equation?
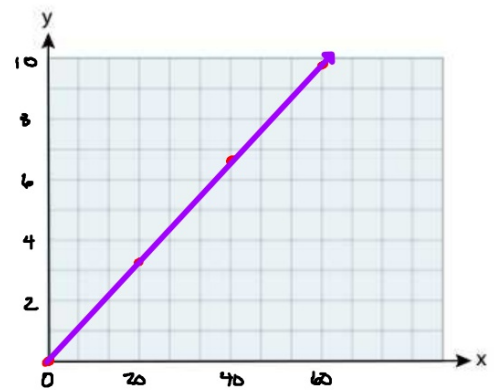
$$y = Kx$$

$$(16.6) = K(100)$$

$$\frac{16.6}{100} = \frac{100K}{100}$$

$$0.166 = K$$

$$y = 0.166x$$

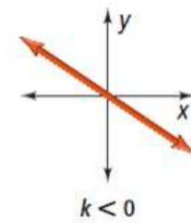| X | Y |
|---|---|
| 0 | $0.166(0) = 0$ |
| 20 | $0.166(20) = 3.32$ |
| 40 | $0.166(40) = 6.64$ |
| 60 | $0.166(60) = 9.96$ |

0-10

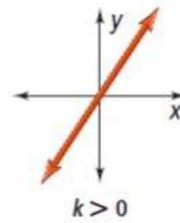**Key Concept:  Graphs of Direct Variations**

The graph of a direct variation equation $y = kx$ is a line with the following properties.
- The line passes through $(0, 0)$.
- The slope of the line is $k$.

$k > 0$

$k < 0$

$k$ = slope

**Example** #4 Writing a Direct Variation From a Table

For the data in the table, does y vary directly with x? If it does, write an equation for the direct variation.

**A**

| X | Y |
|---|---|
| 4 | 6 |
| 8 | 12 |
| 10 | 15 |

$\frac{y}{x}$

$\frac{6}{4} = 1.5$ , $\frac{12}{8} = 1.5$ , $\frac{15}{10} = 1.5$

The ratio $\frac{y}{x} = 1.5$ for each data pair. So y varies directly with x

The direct variation is $\boxed{y = 1.5x}$

**B**

| X | Y |
|---|---|
| −2 | 3.2 |
| 1 | 2.4 |
| 4 | 1.6 |

$\frac{y}{x}$

$\frac{3.2}{-2} = -1.6$ , $\frac{2.4}{1} = 2.4$ , $\frac{1.6}{4} = 0.4$

The ratio $\frac{y}{x}$ is not the same for all data pairs.

So y does not vary directly with x

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4. For the data in the table at the right, does y vary directly with x? If it does, write an equation for the direct variation.

| X | Y |
|---|---|
| −3 | 2.25 |
| 1 | −0.75 |
| 4 | −3 |

$$\frac{2.25}{-3} = -0.75$$

$$\frac{-0.75}{1} = -0.75$$

$$\frac{-3}{4} = -0.75$$

yes it is a direct variation     $y = -0.75x$