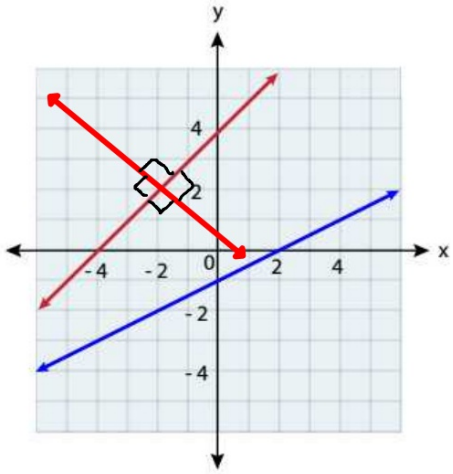
Name _____

5-6 Parallel and Perpendicular Lines

OBJECTIVE: **I can determine whether lines are parallel, perpendicular, or neither yo write equations of parallel lines and perpendicular lines**

Warm-Up

Copy the graph shown at the right. Can you draw a line that will not intersect either of the lines in the graph? If so, draw the line. If not, why not?
Can you draw a line that will intersect one of the lines in such a way that the intersection forms four congruent angles?
If so, draw the line. If not, why not?



Essential Understanding

Essential Understanding You can determine the relationship between two lines by comparing their slopes and y-intercepts.

Key Concept: Slopes of Parallel Lines

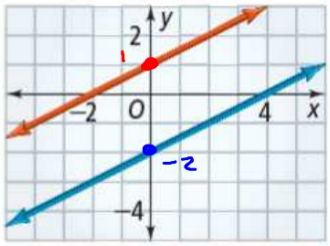
Words

Nonvertical lines are parallel if they have the same slope and different y-intercepts. Vertical lines are parallel if they have different x-intercepts.

Example

The graphs of $y = \frac{1}{2}x + 1$ and $y = \frac{1}{2}x - 2$ are lines that have the same slope, $\frac{1}{2}$, and different y-intercepts. The lines are parallel.

Graph



Example #1 Writing an Equation of a Parallel Line

A line passes through (12, 5) and is parallel to the graph of $y = \frac{2}{3}x - 1$. What equation represents the line in slope-intercept form?

Have the slope → $m_1 = \frac{2}{3}$ point $(12, 5)$, use point slope form

$y - y_1 = m(x - x_1)$

$y - (5) = (\frac{2}{3})(x - (12))$

$y - 5 = \frac{2}{3}(x - 12)$ ← Distribute $\frac{2}{3}$

$y - 5 = \frac{2}{3}x - \frac{2}{3} \cdot 12$

$y - 5 = \frac{2}{3}x - \frac{24}{3}$

$y - 5 = \frac{2}{3}x - 8$ ← Add 5 to both sides

$+5 \qquad +5$

$y = \frac{2}{3}x - 3$

The graph of $y = \frac{2}{3}x - 3$ passes through $(12, 5)$ and is parallel to the graph of $y = \frac{2}{3}x - 1$

Your Turn to Work it Out

1. A line passes through (-3, -1) and is parallel to the graph of y = 2x + 3. What equation represents the line in slope-intercept form?

$m = 2$

Point = $(-3, -1)$

$y - y_1 = m(x - x_1)$

$y - (-1) = (2)(x - (-3))$

$y + 1 = 2(x + 3)$

$y + 1 = 2x + 6$

$-1 \quad\quad -1$

$y = 2x + 5$

$y_1 = 2x + 3$

$y_2 = 2x + 5$

Concept Understanding

You can also use slope to determine whether two lines are perpendicular. Perpendicular lines are lines that intersect to form right angles.

Key Concept: Slopes of Perpendicular Lines

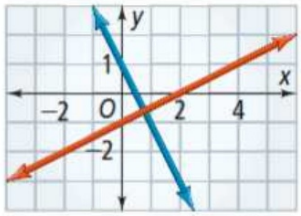
Words

Two nonvertical lines are perpendicular if the product of their slopes is -1. A vertical line and a horizontal line are also perpendicular.

Example

The graph of $y = \frac{1}{2}x - 1$ has a slope of $\frac{1}{2}$.
The graph of $y = -2x + 1$ has a slope of -2.
Since $\frac{1}{2}(-2) = -1$, the lines are perpendicular.

Graph



Two numbers whose product is -1 are opposite reciprocals. So, the slopes of perpendicular lines are opposite reciprocals. To find the opposite reciprocal of $-\frac{3}{4}$, for example, first find the reciprocal, $-\frac{4}{3}$. Then write its opposite, $\frac{4}{3}$. Since $-\frac{3}{4} \# \frac{4}{3} = -1$, $\frac{4}{3}$ is the opposite reciprocal of $-\frac{3}{4}$.

Example #2 Classifying Lines

Are the graphs of $4y = -5x + 12$ and $y = \frac{4}{5}x - 8$ parallel, perpendicular, or neither? Explain.

$\frac{4y}{4} = \frac{-5x}{4} + \frac{12}{4}$

$y_1 = -\frac{5}{4}x + 3$

$m_1 = -\frac{5}{4}$

$y_2 = \frac{4}{5}x - 8$

$m_2 = \frac{4}{5}$

To determine if they are perpendicular, multiply $m_1 \cdot m_2$

$m_1 \cdot m_2 = ?$

$-\frac{5}{4} \cdot \frac{4}{5} = \frac{-20}{20}$

$= -1$ ← this negative indicates that those two functions are perpendicular.

Your Turn to Work it Out

2. Are the graphs of the equations parallel, perpendicular, or neither? Explain.

a. $y = \frac{3}{4}x + 7$ and $4x - 3y = 9$

$y = \frac{3}{4}x + 7$

$m_1 = \frac{3}{4}$

$4x - 3y = 9$

$-4x \quad -4x$

$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{9}{-3}$

$y = \frac{4}{3}x - 3$

$m_2 = \frac{4}{3}$

$m_1 \cdot m_2$

$\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12}$

$= 1$

These graph are neither parallel nor perpendicular

b. $6y = -x + 6$ and $y = -\frac{1}{6}x + 6$

$\frac{6y}{6} = \frac{-x}{6} + \frac{6}{6}$

$y = -\frac{1}{6}x + 1$

$m_1 = -\frac{1}{6}$

$y = -\frac{1}{6}x + 6$

$m_2 = -\frac{1}{6}$

These two graph are parallel.

Example #3 Writing an Equation of a Perpendicular Line

Multiple Choice Which equation represents the line that passes through (2, 4) and is perpendicular to the graph of $y = \frac{1}{3}x - 1$?

(A) $y = \frac{1}{3}x + 10$ (B) $y = 3x + 10$ (C) $y = -3x - 2$ (D) $y = -3x + 10$

$m_1 = \frac{1}{3} \Rightarrow m_2 = -\frac{3}{1}$

Positive? add negative

$y - y_1 = m(x - x_1)$

$y - (4) = (-3)(x - (2))$

$y - 4 = -3(x - 2)$

$y - 4 = -3x + 6$

$+4 \quad +4$

$y = -3x + 10$

Your Turn to Work it Out

3. A line passes through (1, 8) and is perpendicular to the graph of y = 2x + 1. What equation represents the line in slope-intercept form?

$y = 2x + 1$

$m_1 = \frac{2}{1} \Rightarrow m_2 = -\frac{1}{2}$

Positive ? add negative

$y - y_1 = m(x - x_1)$

$y - (8) = (-\frac{1}{2})(x - (1))$

$y - 8 = -\frac{1}{2}(x - 1)$

$y - 8 = -\frac{1}{2}x + \frac{1}{2}$

$+8 \qquad +8$

$y = -\frac{1}{2}x + \frac{17}{2}$.