LESSON 19

Parallel and Perpendicular Lines

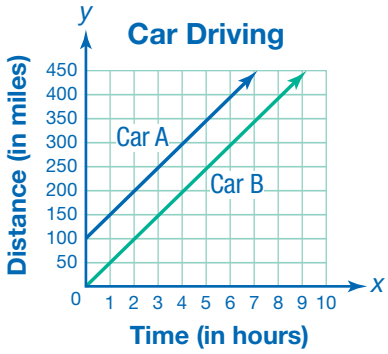
UNDERSTAND **Parallel lines** lie in the same plane but never intersect. On a coordinate plane, lines that are parallel to each other have the same slope but different *y*-intercepts.

Consider two cars traveling in the same direction at the same constant speed. Both cars are traveling at a rate of 50 miles per hour, but Car A is 100 miles ahead of Car B. The following functions describe the distance of each car from Car B's starting point, in miles, after t hours.

Car A: $d_A(t) = 50t + 100$

Car B: $d_B(t) = 50t$

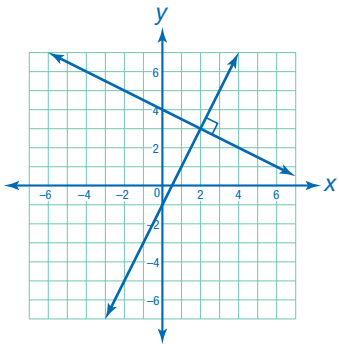
The graphs of these equations are shown on the right.



The equations have the same slope, 50, but different *y*-intercepts. The function for Car A has a *y*-intercept of 100, and the function for Car B has a *y*-intercept of 0. In real terms, at the time when Car B leaves, Car A is 100 miles away. Since the cars travel at the same rate, they will always be 100 miles apart and will never meet, just as the parallel lines in the graph will never intersect.

UNDERSTAND **Perpendicular lines** intersect to form right angles. On a coordinate plane, lines that are perpendicular have slopes that are opposite reciprocals of each other. This means that if one line has a slope of m, then a line perpendicular to it will have a slope of $-\frac{1}{m}$. The product of a number and its opposite reciprocal is always -1.

$$m \cdot -\frac{1}{m} = -1$$



The two lines shown on the right are perpendicular. Use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope of each line.

One line passes through the points (2, 3) and (4, 2).

$$m = \frac{2-3}{4-2} = \frac{-1}{2} = -\frac{1}{2}$$

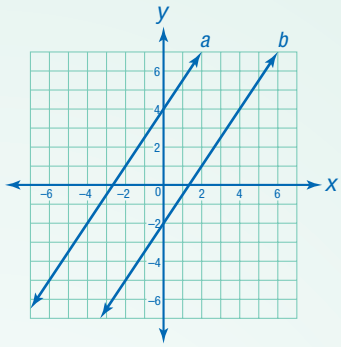
The other line passes through the points (1, 1) and (2, 3).

$$m = \frac{3-1}{2-1} = \frac{2}{1} = 2$$

The negative reciprocal of $-\frac{1}{2}$ is $\frac{2}{1}$, or 2, so the lines are perpendicular.

Connect

Are lines a and b parallel?



1 Observe the lines.

The lines do not intersect at any visible point, so they appear to be parallel. However, the lines may intersect at some point not on the grid, so observation alone is not enough to prove that they are parallel.

2 Find the slope of line a.

Find two points on the line, and use the slope formula to calculate its slope.

Line a passes through the points (0, 4) and (−2, 1).

$$m_a = \frac{1-4}{-2-0} = \frac{-3}{-2} = \frac{3}{2}$$

3 Find the slope of line b.

Line b passes through the points (0, −2) and (2, 1).

$$m_b = \frac{1-(-2)}{2-0} = \frac{3}{2}$$

4 Compare the slopes of lines a and b.

Both lines have a slope of $\frac{3}{2}$. The y-intercept of line a is 4, and the y-intercept of line b is −2. The slopes are the same, and the y-intercepts are different.

▶ Lines a and b are parallel.

TRY

Find the equation of another line that is parallel to lines a and b. Then graph that line on the same coordinate plane.

EXAMPLE A The equations of two lines are shown below. Are lines r and s perpendicular?

Line r: $4y + 12 = x$

Line s: $8y + 2x = 16$

1 Find the slope of line r.

Rewrite the equation in slope-intercept form. The coefficient of x will be the slope.

Solve for y.

$4y + 12 = x$

$4y = x - 12$

$y = \frac{1}{4}x - 3$

The slope is $\frac{1}{4}$.

2 Find the slope of line s.

Rewrite the equation in slope-intercept form. The coefficient of x will be the slope.

Solve for y.

$8y + 2x = 16$

$8y = -2x + 16$

$y = -\frac{1}{4}x + 2$

The slope is $-\frac{1}{4}$.

3 Compare the slopes of lines r and s.

$\frac{1}{4} \cdot \left(-\frac{1}{4}\right) = -\frac{1}{16}$

The product of the slopes is not -1, so the slopes are not opposite reciprocals.

▶ Lines r and s are not perpendicular.

TRY

Determine whether the following lines are perpendicular.

$3y - 2x = 6$

$3x = 14 - 2y$

EXAMPLE B Line p is represented by the equation $2y + 2 = 6x$. Find the following:

- line n, a line that is parallel to line p and that passes through the point (6, 2)
- line q, a line that is perpendicular to line p and that passes through the point (6, 2).

1 Rewrite the equation for line p in slope-intercept form. Find the slope.

Solve for y.

$2y + 2 = 6x$

$2y = 6x - 2$

$y = 3x - 1$

The slope of line p is 3.

2 Find the equation of line n.

A line parallel to line p has the same slope, 3. Use point-slope form to write the equation of a line with a slope of 3 that passes through (6, 2). Then, convert the equation to slope-intercept form.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 6)$

$y - 2 = 3x - 18$

$y = 3x - 16$

3 Find the equation of line q.

A line perpendicular to line p has a slope that is the opposite reciprocal of 3. The opposite reciprocal of 3 is $-\frac{1}{3}$ because $3 \cdot \left(-\frac{1}{3}\right) = -1$.

Use point-slope form to write the equation of a line with a slope of $-\frac{1}{3}$ that passes through (6, 2). Then convert the equation to slope-intercept form.

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{3}(x - 6)$

$y - 2 = -\frac{1}{3}x + 2$

$y = -\frac{1}{3}x + 4$

CHECK

Graph lines p, n, and q on the same coordinate plane. Confirm that line p is parallel to line n and is perpendicular to line q.

Practice

Fill in the blank or write the answer to the question.

1. A line that is parallel to $y = \frac{3}{4}x - 9$ has slope $m =$ _____.

2. A line that is perpendicular to $3y = 11 - 8x$ has slope $m =$ _____.

3. A line that is parallel to $y = 12$ has slope $m =$ _____.

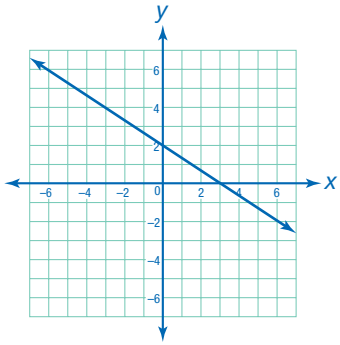
4. Are the lines $2y - x = 6$ and $6x - 3y - 33 = 0$ parallel, perpendicular, or neither?

REMEMBER The slopes of perpendicular lines are opposite reciprocals.

HINT Write the equation of a line in slope-intercept form to find its slope.

Choose the best answer.

5. Which equation represents a line that is perpendicular to the line shown below?



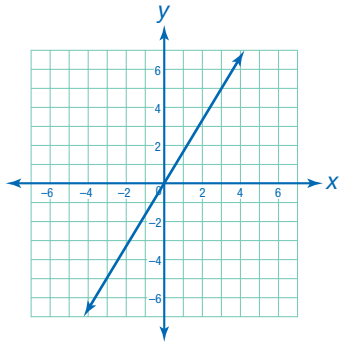
A. $y = \frac{2}{3}x + 5$

B. $y = \frac{3}{2}x - 4$

C. $y = -\frac{2}{3}x - 6$

D. $y = -\frac{3}{2}x + 1$

6. Which equation represents a line that is parallel to the line shown below?



A. $y = \frac{3}{5}x + 1$

B. $y = -\frac{3}{5}x + 1$

C. $y = \frac{5}{3}x - 1$

D. $y = -\frac{5}{3}x - 1$

7. Which describes the lines $y = \frac{7}{8}x + 12$ and $y = -\frac{8}{7}x + 7$?

A. parallel

B. perpendicular

C. neither parallel nor perpendicular

8. Which describes the lines $x - 2y = -6$ and $4y + 4 = 2x$?

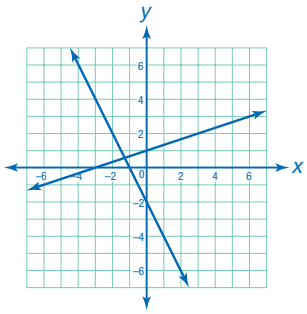
A. parallel

B. perpendicular

C. neither parallel nor perpendicular

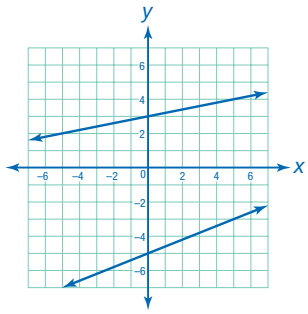
Choose the best answer.

9. Which describes the lines shown below?



A. parallel
B. perpendicular
C. neither parallel nor perpendicular

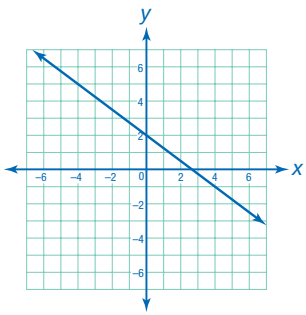
10. Which describes the lines shown below?



A. parallel
B. perpendicular
C. neither parallel nor perpendicular

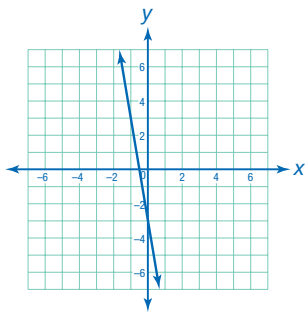
Write the equation of the line that is described. Give your answer in slope-intercept form.

11.



A line that is parallel to the one shown above and that passes through the point (8, −7).

12.



A line that is perpendicular to the one shown above and that passes through the point (12, 3).

13. A line that is parallel to $3y = x + 12$ and that passes through the point (6, −8).

14. A line that is perpendicular to $y - x = 7$ and that passes through the point (−2, −2).

Solve.

15. **EXPLAIN** Lines *s*, *t*, and *u* all lie on the same plane. Line *s* is parallel to line *t*. Line *t* is perpendicular to line *u*. What is the relationship between lines *s* and *u*? How do you know?