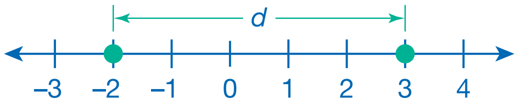
LESSON

20 Distance in the Plane

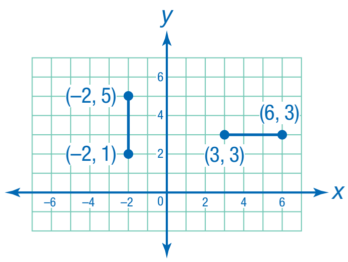
UNDERSTAND It's easy to calculate the distance between two points on a number line.



The distance is equal to the difference of the two numbers.

$d = 3 - (-2) = 5$

It's just as easy to calculate the length of a vertical or horizontal line segment on the coordinate plane.

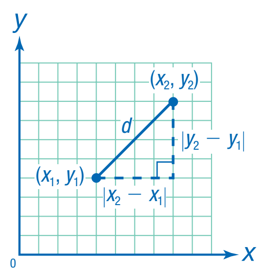


For a vertical line segment, the *x*-coordinates of the endpoints are the same. So, the length of the line segment is simply the difference of the *y*-coordinates.

$d = 5 - 1 = 4$

For a horizontal line segment, the *y*-coordinates of the endpoints are the same. So, the length of the line segment is simply the difference of the *x*-coordinates.

$d = 6 - 3 = 3$



Finding the length of a line segment that is not horizontal or vertical is trickier. Recall the **Pythagorean Theorem**, which states that, for any right triangle with legs of length a and b and hypotenuse of length c, $a^2 + b^2 = c^2$. You can think of a diagonal line on the coordinate plane as the hypotenuse of a triangle with one vertical leg and one horizontal leg.

The horizontal leg has a length of $|x_2 - x_1|$. The vertical leg has a length of $|y_2 - y_1|$. You can substitute these expressions into the Pythagorean Theorem and solve for d, the length of the diagonal line.

$$a^2 + b^2 = c^2$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

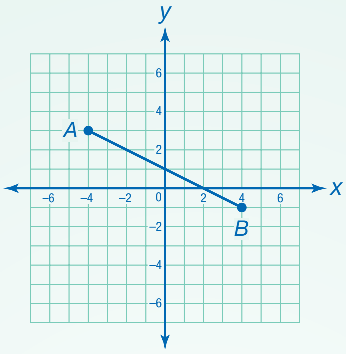
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is called the distance formula. It can be used to find the length of any line segment on the coordinate plane, as long as its endpoints are known.

Connect

The coordinate plane shows point A, point B, and the line segment connecting them.



Use the distance formula to find AB, the length of the line segment.

1 Find the coordinates of the endpoints.

Point A is located at $(-4, 3)$.

Point B is located at $(4, -1)$.

Let $A(-4, 3) = (x_1, y_1)$ and let $B(4, -1) = (x_2, y_2)$.

2 Apply the distance formula.

Substitute the coordinates into the formula and evaluate the radicand.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - (-4))^2 + (-1 - 3)^2}$$
$$= \sqrt{(8)^2 + (-4)^2}$$
$$= \sqrt{64 + 16}$$
$$= \sqrt{80}$$

3 Determine if the result can be simplified further.

The radicand, 80, is not a perfect square. However, it has factors that are perfect squares. Simplify by factoring out any perfect square factors.

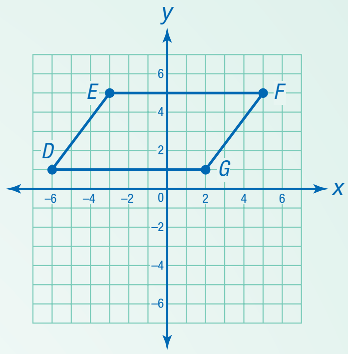
$$d = \sqrt{80}$$
$$d = \sqrt{16 \cdot 5}$$
$$d = \sqrt{16} \cdot \sqrt{5}$$
$$d = 4\sqrt{5}$$

TRY

Substitute the points in the reverse order: Let $B(4, -1) = (x_1, y_1)$ and let $A(-4, 3) = (x_2, y_2)$. Do you get the same result? Why do you think this is?

EXAMPLE A Parallelogram *DEFG* is shown on the coordinate plane.

What is the perimeter of parallelogram *DEFG*?



1 Determine what lengths to find.

Recall that opposite sides of a parallelogram are congruent. So, you only need to find the lengths of two adjacent sides. Find the lengths of $\overline{DE}$ and $\overline{EF}$.

2 Find the length of $\overline{EF}$.

The coordinates of the endpoints of $\overline{EF}$ are $E(-3, 5)$ and $F(5, 5)$. Since $\overline{EF}$ is horizontal, you do not need to use the distance formula. The *y*-coordinates are the same. To find the length, find the absolute value of the difference of the *x*-coordinates.

$$EF = |-3 - 5| = |-8| = 8$$

Opposite sides of a parallelogram are congruent, so $GD = EF$.

$$GD = EF = 8$$

3 Find the length of $\overline{DE}$.

The coordinates of the endpoints of $\overline{DE}$ are $D(-6, 1)$ and $E(-3, 5)$. Since $\overline{DE}$ is diagonal, use the distance formula. Let $D(-6, 1) = (x_1, y_1)$ and $E(-3, 5) = (x_2, y_2)$.

$$DE = \sqrt{(-3 - (-6))^2 + (5 - 1)^2}$$

$$DE = \sqrt{(3)^2 + (4)^2}$$

$$DE = \sqrt{9 + 16}$$

$$DE = \sqrt{25}$$

$$DE = 5$$

Opposite sides of a parallelogram are congruent, so $FG = DE$.

$$FG = DE = 5$$

4 Find the perimeter.

$$P = DE + EF + FG + GD$$

$$P = 5 + 8 + 5 + 8$$

$$P = 26$$

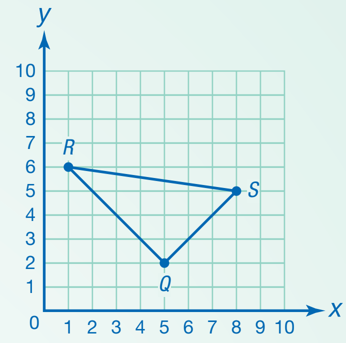
▶ The perimeter of parallelogram *DEFG* is 26 units.

DISCUSS

Imagine a regular octagon in a coordinate plane. How many side lengths would you need to find in order to calculate its perimeter?

EXAMPLE B Right triangle *QRS* is shown on the coordinate plane.

Find the area of $\triangle QRS$.



1 Determine what lengths to find.

$\triangle QRS$ is a right triangle with the right angle at $\angle Q$. In a right triangle, the legs form the base and the height. So, find *QR* and *QS*.

2 Find the length of $\overline{QR}$.

Let $Q(5, 2) = (x_1, y_1)$ and $R(1, 6) = (x_2, y_2)$.

$QR = \sqrt{(1 - 5)^2 + (6 - 2)^2}$

$QR = \sqrt{(-4)^2 + (4)^2}$

$QR = \sqrt{32}$

$QR = 4\sqrt{2}$

3 Find the length of $\overline{QS}$.

Let $Q(5, 2) = (x_1, y_1)$ and $S(8, 5) = (x_2, y_2)$.

$QS = \sqrt{(8 - 5)^2 + (5 - 2)^2}$

$QS = \sqrt{(3)^2 + (3)^2}$

$QS = \sqrt{18}$

$QS = 3\sqrt{2}$

4 Find the area of $\triangle QRS$.

The area of a triangle is half of the base times the height. Let $\overline{QR}$ be the base and let $\overline{QS}$ be the height.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}QR \cdot QS$

$A = \frac{1}{2}(4\sqrt{2})(3\sqrt{2})$

$A = 12$

▶ The area of $\triangle QRS$ is 12 square units.

TRY

Find the area of $\triangle XYZ$ with vertices $X(-4, 2)$, $Y(2, 2)$ and $Z(-1, 5)$.

Practice

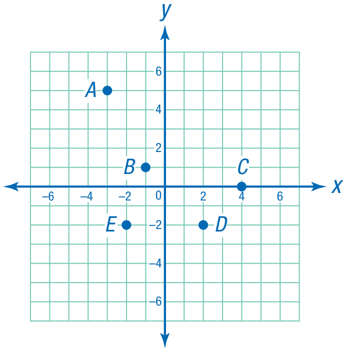
Use the coordinate plane below for questions 1–4. Find the distance in units between each given pair of points and write it in simplest form.

1. *D* and *E* ________________

2. *A* and *C* ________________

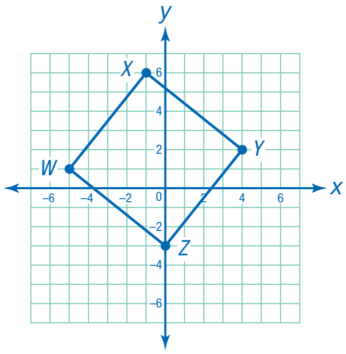
3. *B* and *D* ________________

4. *A* and *E* ________________



Use the information below for questions 5 and 6. Choose the best answer.

Figure *WXYZ* on the coordinate plane below is a square.



5. What is the perimeter of *WXYZ*?
   A. $2\sqrt{41}$ units
   B. 20 units
   C. $4\sqrt{39}$ units
   D. $4\sqrt{41}$ units

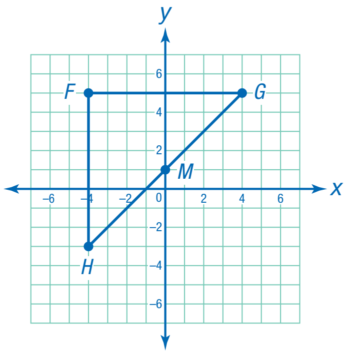
6. What is the area of *WXYZ*?
   A. 25 units^2
   B. 39 units^2
   C. 41 units^2
   D. 82 units^2

Solve.

7. The distance between points *A* and *B* is $\sqrt{113}$. Point *A* is located at $(-3, 6)$, and point *B* is located at $(4, y)$. What is a possible value of *y*? ____________

8. The distance between points *C* and *D* is $6\sqrt{2}$. Point *C* is located at the origin. Point *D* is located at the point (a, a). What is a possible value of *a*? ____________

9. Triangle *FGH* is isosceles with base $\overline{GH}$. Point *M* is the midpoint of $\overline{GH}$.



Find the length of altitude $\overline{FM}$, the perimeter of $\triangle FGH$, and the area of $\triangle FGH$.

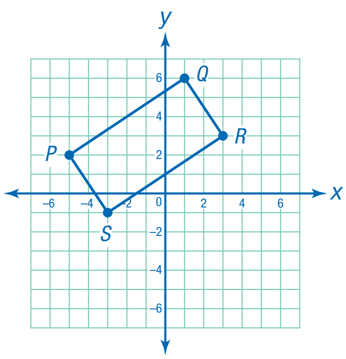
Altitude: ____________________

Perimeter: ____________________

Area: ____________________

Use the information below to answer questions 10 and 11.

Rectangle *PQRS* is shown on the coordinate plane below.



10. **PLAN** How can you find the area of rectangle *PQRS*?

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11. **APPLY** Find the area of rectangle *PQRS*.

Area: ____________________