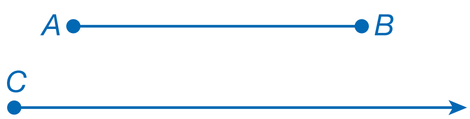
LESSON 8

Constructions of Lines and Angles

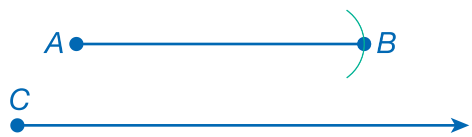
An important part of studying geometry is learning how to **construct** certain basic geometric figures. Some of the figures you can construct are line segments, angles, parallel lines, and perpendicular lines. Some of the tools you may use are a compass and a straightedge.

EXAMPLE A Construct a line segment that is congruent to $\overline{AB}$.

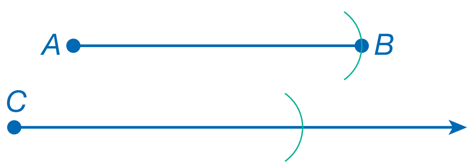
1 Using your straightedge, draw a ray that is longer than $\overline{AB}$. Label the endpoint as point C.



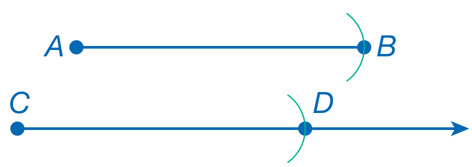
2 Place the compass point on point A. Place the pencil tip on point B. Then draw a curve.



3 Without adjusting the compass span, place the compass point on point C. Draw a curve through the ray.



4 Label the point where the curve intersects the ray as point D.

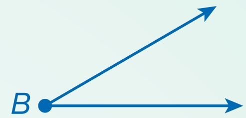


$\overline{CD}$ is congruent to $\overline{AB}$.

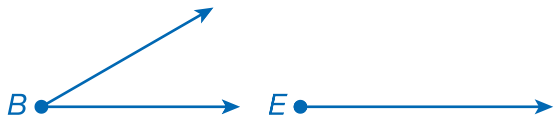
DISCUSS

How could you construct a line segment that is twice as long as $\overline{AB}$?

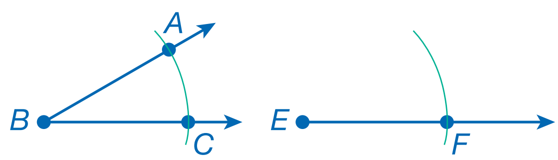
EXAMPLE B Construct an angle that is congruent to $\angle B$.



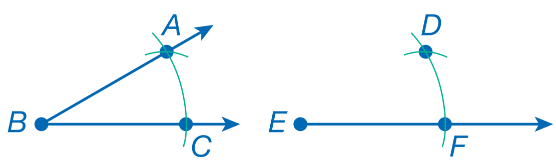
1 Using a straightedge, draw a ray. Label the endpoint as point *E*.



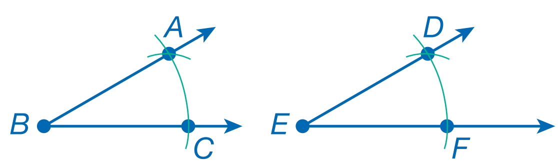
2 Place the compass point on point *B* and draw a curve. Label the points of intersection *A* and *C*. Then, without changing the compass span, place the compass point on point *E*. Draw a curve through the ray. Be sure that the curve extends well above the ray. Label the point of intersection as point *F*.



3 Place the compass point on point *C* and the pencil tip on point *A*. Draw a curve through point *A*. Then, without changing the compass span, place the compass point on *F*. Draw a curve that intersects the curve you drew earlier. Label the point where the two curves intersect as point *D*.



4 Draw a ray from point *E* through point *D*.

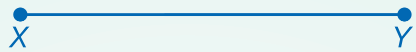


▶ $\angle E$ is congruent to $\angle B$.

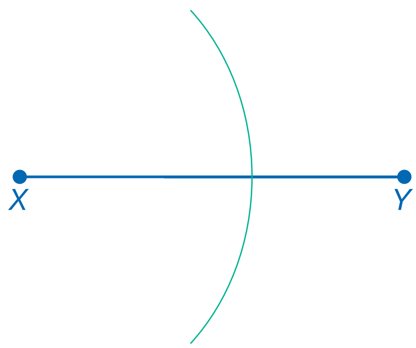
DISCUSS

How could you construct an angle with twice the measure of $\angle B$?

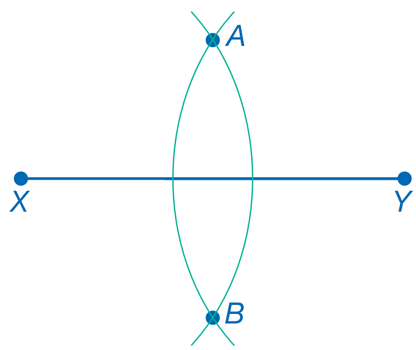
EXAMPLE C Construct a line that bisects $\overline{XY}$.



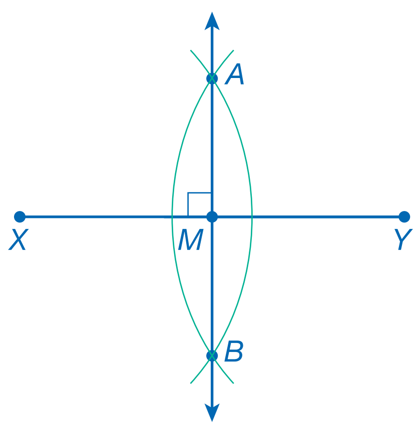
1 Place the compass point on point *X*. Adjust the compass span so that it is more than half the length of *XY*. Draw a curve that intersects $\overline{XY}$.



2 Without adjusting the compass span, place the compass point on point *Y*. Draw a second curve that intersects the first curve in two places. Label the points of intersection as points *A* and *B*.



3 Use a straightedge to draw a line through points *A* and *B*. Label the intersection of $\overline{XY}$ and $\overleftrightarrow{AB}$ as point *M*.

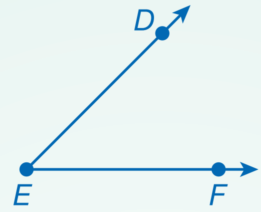


▶ Point *M* is the midpoint of $\overline{XY}$, and $\overleftrightarrow{AB}$ bisects $\overline{XY}$.

DISCUSS

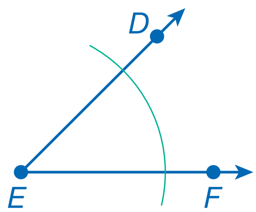
What can be said about the lengths of $\overline{AM}$ and $\overline{BM}$?

EXAMPLE D Construct the bisector of $\angle DEF$.



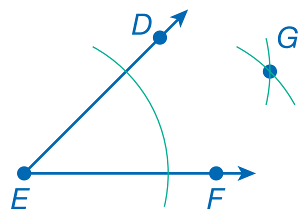
1

Place the compass point on the vertex of the angle, point E. Draw a curve that intersects both $\overrightarrow{ED}$ and $\overrightarrow{EF}$.



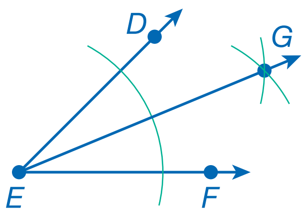
2

Place the compass point at the intersection of the curve and $\overrightarrow{ED}$. Draw a small curve inside the angle. Without changing the compass span, place the compass point at the intersection of the first curve and $\overrightarrow{EF}$. Draw another small curve inside the angle so that it intersects the small curve you drew earlier. Label the intersection of the curves as point G.



3

Use the straightedge to draw a ray from point E through point G.



▶ Ray EG is the **angle bisector** of $\angle DEF$.

TRY

On a separate sheet of paper, draw a line segment and construct its bisector. Then bisect one of the angles formed by the line segment and its bisector.

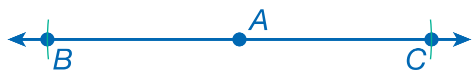
EXAMPLE E Construct a perpendicular line from a point on a line.

1

Draw a line. Then draw point *A* on the line.

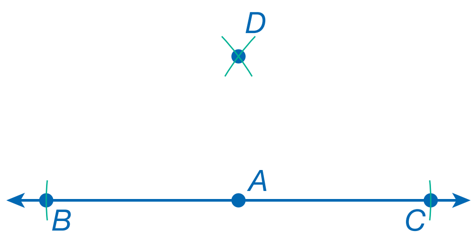
2

Place the compass point on point *A*. Set the compass span to any width. Draw a curve that intersects the line to the left of point *A*. Label the point of intersection as point *B*. Without adjusting the compass span, draw another curve to the right of point *A*. Label the point of intersection as point *C*.



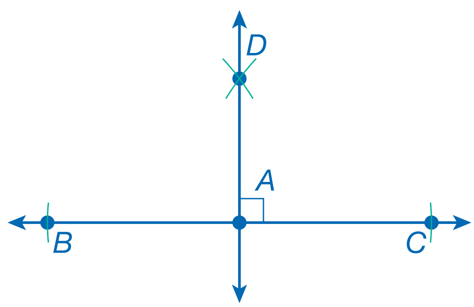
3

Place the compass on point *B*. Set the compass span to any width greater than the length of $\overline{AB}$. Draw a curve above point *A*. Without adjusting the compass span, place the compass point on point *C*, then draw a curve that intersects the arc you just drew. Label the point of intersection as point *D*.



4

Use your straightedge to draw a line that connects points *A* and *D*.



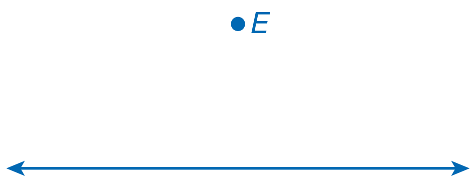
▶ $\overleftrightarrow{AD}$ is perpendicular to $\overleftrightarrow{BC}$ at point *A*.

DISCUSS

How could you construct a perpendicular line from point *B*?

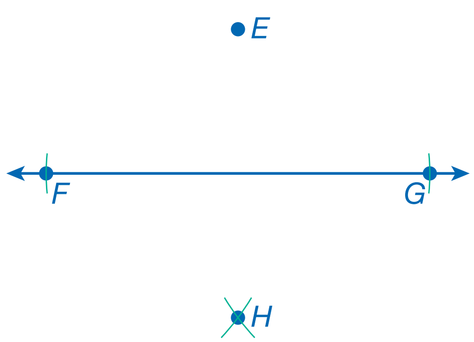
EXAMPLE F Construct a perpendicular line from a point off a line.

1 Draw a line. Then draw point *E* above the line.

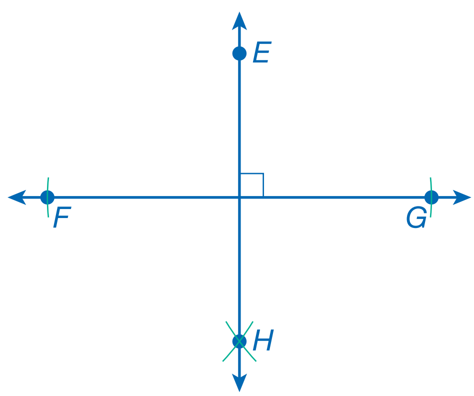


2 Place the compass point on point *E*. Set the compass span to a width greater than the distance from point *E* to the line. Draw a curve that intersects the line to the left of point *E*. Label the point of intersection as point *F*. Without adjusting the compass span, draw another curve that intersects the line to the right of point *E*. Label the point of intersection as point *G*.

3 Place the compass point on point *F*. Without adjusting the compass span, draw a curve below the line. Now place the compass point on point *G*. Then again without adjusting the compass span, draw a curve that intersects the curve below the line. Label the point of intersection as point *H*.



4 Use a straightedge to draw a line connecting points *E* and *H*.

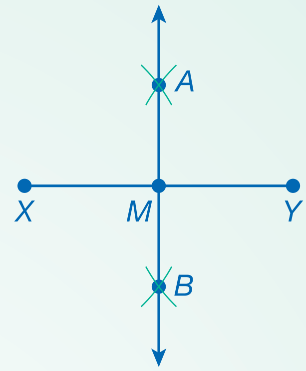


▶ $\overleftrightarrow{EH}$ is perpendicular to $\overleftrightarrow{FG}$.

TRY

In this example, if point *E* were below the original line, how would that change the process you use to construct a perpendicular line from point *E*?

EXAMPLE G In Example C, you constructed $\overleftrightarrow{AB}$, the bisector of $\overline{XY}$. Prove that $\overleftrightarrow{AB}$ is perpendicular to $\overline{XY}$.



1 Draw triangles *AMX* and *AMY*. Use the SSS Postulate to prove the triangles are congruent.

Draw segments *AX* and *AY* to form triangles *AMX* and *AMY*.

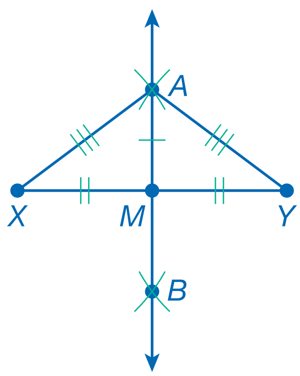
Use the relationship between corresponding sides of the triangles to show that $\triangle AMX \cong \triangle AMY$.

The two triangles share side $\overline{AM}$.

$\overline{XM} \cong \overline{YM}$ because $\overleftrightarrow{AB}$ bisects $\overline{XY}$ at point *M*.

$\overline{AX} \cong \overline{AY}$ because these distances were drawn with the same compass span.

So, $\triangle AMX \cong \triangle AMY$ by the SSS Postulate.



2 Show that $\overleftrightarrow{AB}$ is perpendicular to $\overline{XY}$.

$\angle AMX \cong \angle AMY$ because corresponding angles of congruent triangles are congruent.

$m\angle AMX = m\angle AMY$

Angles *AMX* and *AMY* are a linear pair, so they are supplementary. The sum of their measures is 180°.

$180° = m\angle AMX + m\angle AMY$

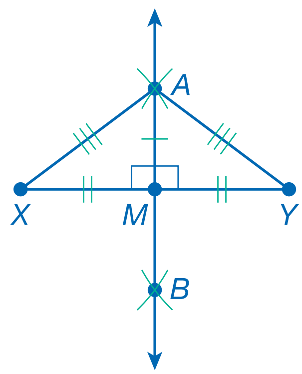
Substitute $m\angle AMX$ for $m\angle AMY$.

$180° = m\angle AMX + m\angle AMX = 2 \cdot m\angle AMX$

$90° = m\angle AMX$

$m\angle AMY = m\angle AMX = 90°$

The angles formed by the intersection of $\overline{XY}$ and $\overleftrightarrow{AB}$ are right angles.

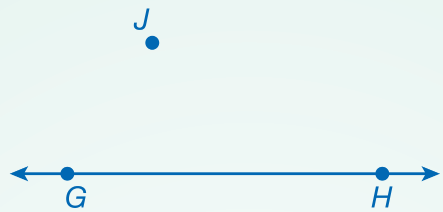


▶ $\overleftrightarrow{AB}$ is the **perpendicular bisector** of $\overline{XY}$.

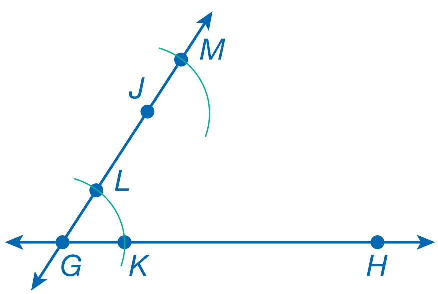
DISCUSS

Does a line have a perpendicular bisector?

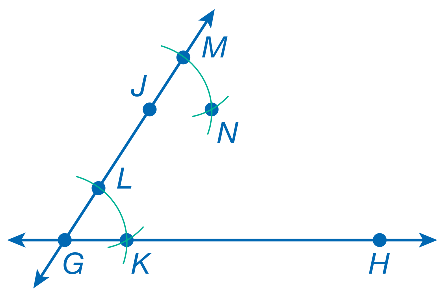
EXAMPLE H Construct a line parallel to $\overleftrightarrow{GH}$ through point *J*.



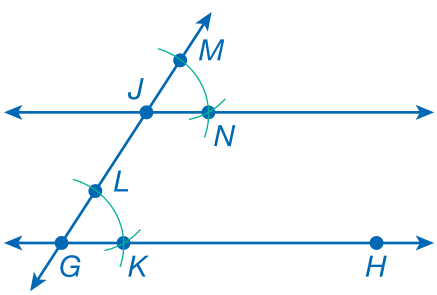
1 Use a straightedge to draw $\overleftrightarrow{GJ}$. Place the compass point on point *G*. Draw a small curve that intersects $\overleftrightarrow{GH}$ and $\overleftrightarrow{GJ}$. Label the points of intersection as points *K* and *L*. Then, without adjusting the compass span, place the compass point on point *J*. Draw a curve that intersects $\overleftrightarrow{GJ}$ above point *J*. Label that point of intersection as point *M*.



2 Place the compass point on point *L*. Adjust the compass span to draw a curve through point *K*. Then, without adjusting the compass span, place the compass point on point *M*. Draw a small curve in the interior of the angle that intersects the curve you drew earlier from point *J*. Label the point where those curves intersect as point *N*.



3 Use a straightedge to draw a line through points *J* and *N*.



▶ Line *JN* is parallel to $\overleftrightarrow{GH}$.

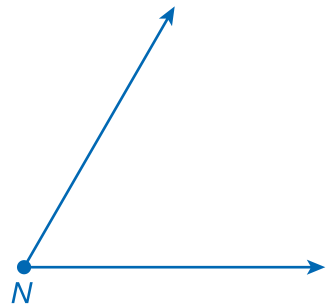
DISCUSS

How does the construction above relate to Example B, in which you copied an angle?

Practice

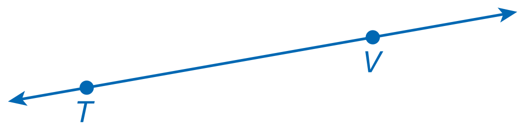
1. Construct a line segment congruent to $\overline{LM}$. Label the new segment $\overline{NP}$.

2. Construct the bisector of $\angle N$. Label the bisector $\overrightarrow{NQ}$.



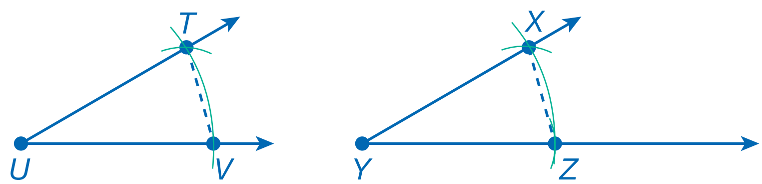
3. Construct a line perpendicular to $\overleftrightarrow{QP}$ at point P. Label the line $\overleftrightarrow{RS}$.

4. Construct a line parallel to $\overleftrightarrow{TV}$. Label the line $\overleftrightarrow{WZ}$.



5. Bisect segment *LM*. Label the bisector $\overleftrightarrow{AB}$. Label the point where $\overleftrightarrow{AB}$ intersects $\overline{LM}$ as point *N*.

6. **THINK CRITICALLY** Think about the steps used to construct an angle congruent to a given angle.



How can you use the steps for copying an angle and the triangle congruence postulates and theorems to prove that the angles are congruent?

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